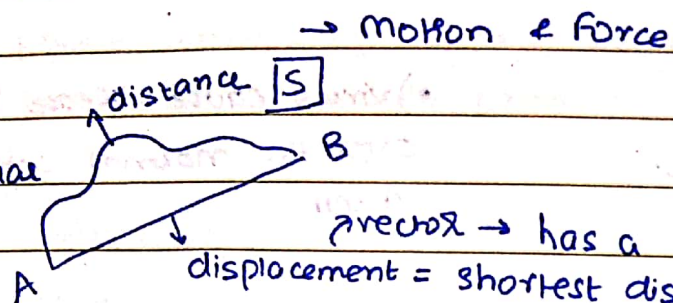


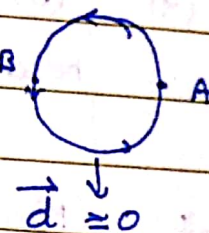
Unit = meter

scalar
bcz has no
specific
direction



for a moving body $S \neq 0$

($d=0$) but displacement can be zero if it is closed path because starting & ending point is same



for translational motion (linear or straight path)

$$S = d$$



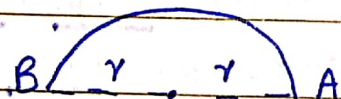
$$\frac{S}{d} = 1$$

otherwise $S > d$

The ratio of magnitude of displacement to distance (less than 1)

$$\frac{d}{S} \leq 1$$

→ for

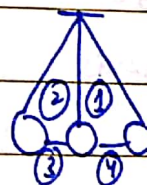


$$S = 2\pi r = \pi r$$

2

$$\vec{d} = 2r$$

for
1 complete
oscillation



$\vec{d} = 0$ because it is a closed path

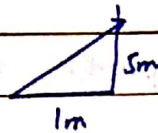
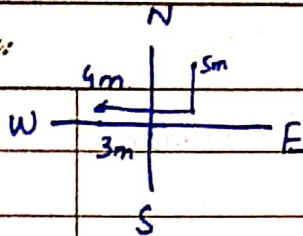
S would 4 times of its
amplitude

Yousaf

$\vec{a}, \vec{v}$ same direction

Day: / /

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$$\vec{d} = \sqrt{(1)^2 + (5)^2} = \sqrt{26} \text{ m}$$

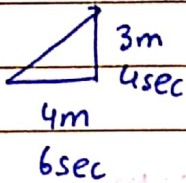
→ Mostly vectors on the same line are discussed like here W/E / N/S

Speed → time rate of change of distance $v = \Delta s / \Delta t$ (ms^{-1}) scalar

velocity → time rate of change of displacement $\vec{v} = \vec{d} / t = \text{ms}^{-1}$

$\vec{v}_{av} = 0$ for closed path $\vec{v}_{av} \neq 0$ not a closed path

Q →



$$v_{av} = \frac{3+4}{4+6} = \frac{7}{10} = 0.7 \text{ ms}^{-1}$$

$$\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t} = \frac{\sqrt{(3)^2 + (4)^2}}{10} = \frac{\sqrt{9+16}}{10} = \frac{\sqrt{25}}{10} = \frac{5}{10} = \frac{1}{2} \text{ ms}^{-1}$$

↓
perpendicular

A FORMULAS FOR AVERAGE SPEED

①

if $s_1 = s_2$ but $t_1 \neq t_2$

$$v_{av} = \frac{2(v_1)(v_2)}{v_1 + v_2} = \frac{2(60)(40)}{60 + 40} = \frac{4800}{100} = 48 \text{ kmh}^{-1}$$

$$\frac{120}{40} = 3$$

$$\frac{60}{40} = 1.5$$

$$\frac{3}{1.5} = 2$$

$$\frac{4800}{100} = 48$$

e.g. 60 kmh^{-1} 40 kmh^{-1} $s = 300 \text{ m. on both side}$ $\therefore t_1 \neq t_2$

$\vec{v}_{av}$ (average displacement) = 0 (bcz it is a closed path)

②

if $s_1 \neq s_2$ but $t_1 = t_2$

$$v_{av} = \frac{v_1 + v_2}{2}$$

③

$s_1 = s_2$ & $t_1 = t_2$

$v_{av} = v_{ins}$ because was moving with average or uniform speed.

Yousaf

Day: / /

Date: / /

- (4) $s_1 \neq s_2, t_1 \neq t_2$ average speed of different conditions

$$V_{av} = \frac{s_1 + s_2}{t_1 + t_2}$$

→ Acceleration: rate of change of velocity

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{ms^{-1}}{s} = ms^{-2}$$

$$\vec{v} = \frac{d}{t} = ms^{-1}$$

- (1) In the case of Increasing velocity on straight line

$$\theta = 0^\circ \quad \vec{a} \rightarrow \vec{v} \quad \text{parallel}$$

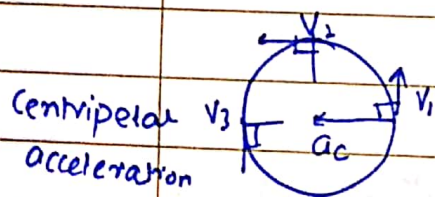
- (2) In case of decreasing velocity (deacceleration)

$$\theta = 180^\circ \quad \vec{a} \leftarrow \vec{v} \quad \text{antiparallel}$$

- (3) $\vec{v} = 0$ $a = 0$ because velocity acceleration is the rate of change of velocity ($a = \text{constant} \rightarrow \text{rare case}$) **MLQ**

→ It is possible that if the speed all over the closed path is constant but velocity is not constant **Yes**

In a circle



→ a_c towards centre so $\theta = 90^\circ$
→ velocity along tangent

velocity $\neq$ constant

imp
MLQ

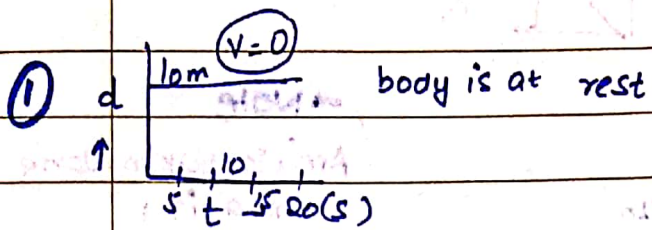
Speed = $v_1 = v_2 = v_3 = \text{constant}$

Day:

Graphs

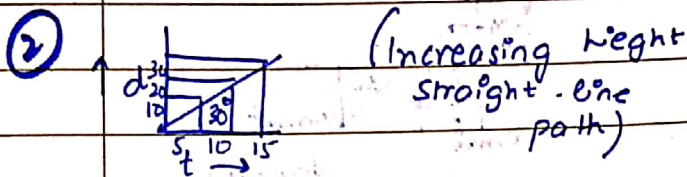
Date: / /

Distance - Time Graph



gives info about average speed
 $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{0}{5} = 0$
 $\Delta x = 5$ $\Delta t = 5$
 $V_{av} = 0$

horizontal straight line graph



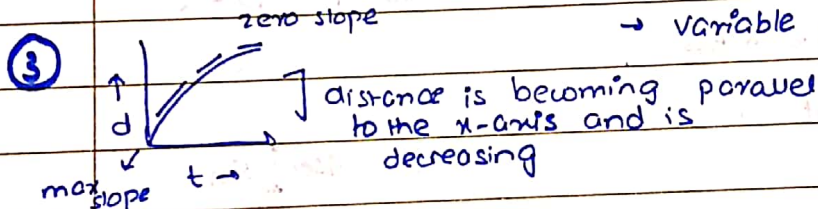
→ particle is moving with constant uniform speed with uniform intervals.

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{30}{15} = 2 \text{ ms}^{-1}$$

$$\text{slope} = \tan \phi = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

if some angle is 30° then

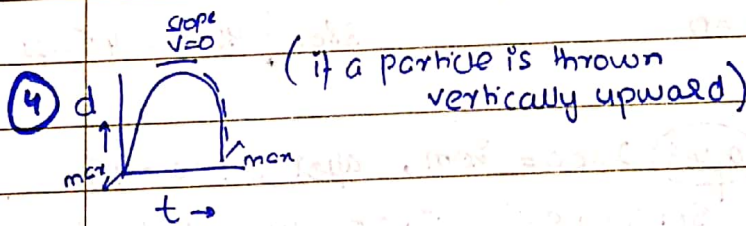
tangent would be found



→ variable speed in time intervals

→ decreasing velocity graph

decreasing velocity - graph

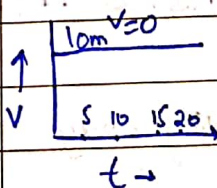


Velocity - Time Graph

→ gives info about

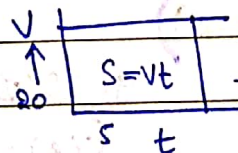
• area of v-t graph = distance

$$\text{slope} = \frac{\Delta v}{\Delta t} = a = \tan \theta = (\vec{a})$$



(Area = distance)

→ body at rest



→ in the case of rectangle

$$\text{Area} = S = 20 \times 5 = 100 \text{ ms}^{-1}$$

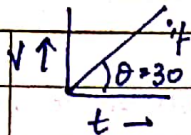
Yousaf

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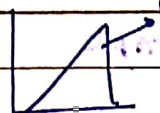
constant acceleration

triangle in case of

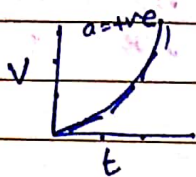
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$$\tan \theta = a$$



$$\frac{1}{2} vt = S \rightarrow \text{distance covered}$$

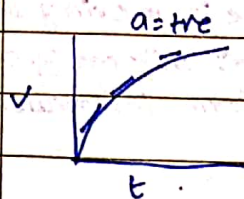


acceleration also increasing at every point

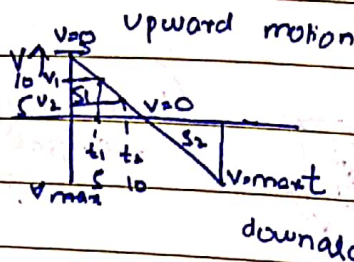
Note
(Area is taken along x-axis)

(non uniform)

increasing velocity graph



Acceleration increases and decreases at slope

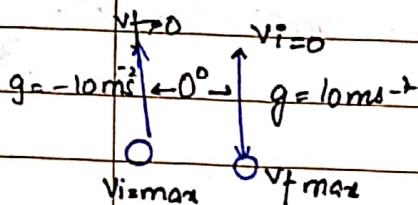


particle thrown vertically upward

→ For distance calculation = $S_1 + S_2$

→ for displacement = $S_1 - S_2$

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{5 - 10}{10 - 5} = \frac{-5}{5} = -1 \text{ ms}^{-2}$$



because the diff in sign is due to the reference from initial point

→ Distance covered by a freely falling body
 $S = \frac{v_i t}{2} + \frac{1}{2} g t^2$ $v_i = 0$

1 sec	5m	
2 sec	20m	$20 - 5 = 15\text{m}$
3 sec	45m	$45 - 20 = 25\text{m}$

distance after 1 sec = 5m

(after) 2 sec = 20m, after 3 sec = 45m

Ratio of the distance

$$S_1 : S_2 : S_3 = 5 : 20 : 45, 1 : 4 : 9$$

→ Ratio of the distance (in) 1 sec, 2 sec, 3 sec : $5 : 15 : 25 \rightarrow 1 : 3 : 5$

→ Distance covered by the n^{th} second of a freely falling body

$$S_n = \frac{g}{2} (2n - 1)$$

e.g. $S_1 = \frac{10}{2} (2(1) - 1)$
 $S(1) = 5\text{m}$

$\frac{9}{2} (2(2) - 1)$
 $S(3) = 15\text{m}$

Yousaf

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Velocities		Degree	rad	$\sin \theta$	$\cos \theta$	$\tan \theta$
Elastic collision (after collision)	$V_1' = \frac{(m_1 - m_2)}{m_1 + m_2} V_1 + \frac{2m_2 V_2}{m_1 + m_2}$	0°	0	0	1	0
		30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
	$V_2' = \frac{2m_1 V_1}{m_1 + m_2} + \frac{(m_2 - m_1) V_2}{m_1 + m_2}$	45°	$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1
		60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
		90°	$\pi/2$	1	0	not defined

① $m_1 = m_2$ (identical masses)

then the velocities would be interchanged

*) after collision $\rightarrow V_1' = V_2, V_2' = V_1$

the velocity of the first particle would be equal to the velocity of the second particle before collision and vice versa.

③* $m_1 = m_2$, second object is at rest $V_2 = 0$

so $V_1' = 0, V_2' = V_1$

② $m_1 \ll m_2, V_2 = 0$ (light particle collides with the heavy object at rest)

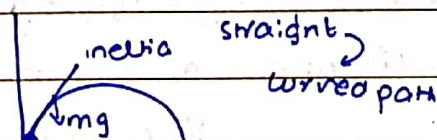
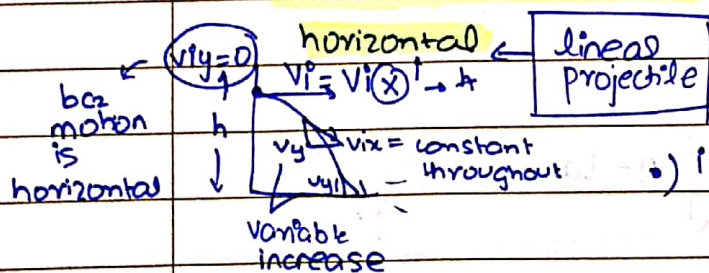
$\rightarrow V_1' = -V_1$ (as a light particle will rebound with the same speed)
 $\rightarrow V_2' = 0$ in opposite direction

④ $m_1 \gg m_2$ (a heavy object collides with a light stationary particle)

$V_1' = V_1$ (no change) in magnitude & direction

$V_2' = 2V_1$

PROJECTILE MOTION



Ignored air resistance

$F_x = 0$

$a_x = 0$

then $V_x = \text{constant}$

then $F_y = (mg) \rightarrow \text{weight}$

$a_y = g$

$F_y = \text{variable}$

Yousaf

path of projectile $\rightarrow$ trajectory

shape $\rightarrow$ parabolic /

• Convention

upward $\rightarrow$ +ve

downward $\rightarrow$ -ve ($a_y = -g$)

Day:

magnitude of

$\rightarrow$ To determine distance Velocity

(variable due to v_y) $V = \sqrt{(V_{px})^2 + (V_{py})^2}$ Variable

$\rightarrow$ To determine distance

•) X - distance.

$$S = v_i t + \left[\frac{1}{2} a t^2 \right] = 0$$

as $a_x = 0$

so $x = v_{ix} t$

•) Y distance

$$S = v_i t + \left[\frac{1}{2} g t^2 \right]$$

$\rightarrow v_i = 0$

so $y = \frac{1}{2} g t^2$

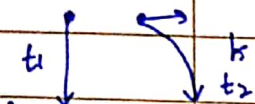
•) $t = \sqrt{\frac{2y}{g}}$

throw horizontally

drop downward

Q) which one will reach ground first?

$t_2 = t_1$ $\rightarrow$ by using $t = \sqrt{\frac{2y}{g}}$ in this we see that both will drop at same time.



if it is thrown downward, now $t_1 > t_2$

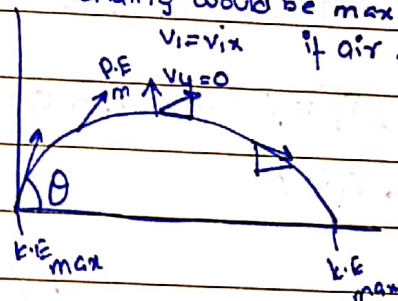
•) oblique projection

ignore air resistance

$\rightarrow F_x = 0, a_x = 0, v_x = \text{constant}$

$\rightarrow F_y = \text{weight}, a_y = (-g) \rightarrow$ from v-t graph

$\rightarrow$ t.E equal at launching & landing would be max and equal if air -2 is ignored



• velocity $v_f = v_i + at = v_{fy} = v_{iy} - gt$ ($v_{iy} \neq 0$ because in oblique projection velocity has components)

$\rightarrow$ vertical $v_{fy} = v_{ix} - gt$

$\rightarrow$ horizontal as $a_x = 0$ so

$v_{fx} = v_{ix}$

$$v_f = \sqrt{(v_{fx})^2 + (v_{fy})^2}$$

$$\theta = \tan^{-1} \frac{v_{fy}}{v_{fx}}$$

Day: / /

Date: / /

$$K.E \text{ at highest point} = K.E_i \times \cos^2 \theta$$

$$P.E \text{ at } \dots = K.E_i \times \sin^2 \theta$$

$$K.E \rightarrow \theta = 45^\circ \quad K.E_i = K$$

$$= K (\sin^2 45)$$

$$K \left(\frac{1}{2} \right)$$

$$= K/2$$

$$K.E_{\text{Peak}} = 200 \times \cos^2 \theta$$

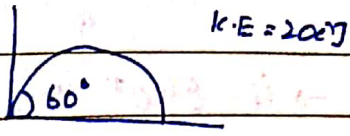
$$\frac{200 \times 100}{2} = 10000$$

$$P.E = K.E_i \times \sin^2 \theta$$

$$= 200 \times \sin^2 60^\circ$$

$$200 \times \left(\frac{\sqrt{3}}{2} \right)^2$$

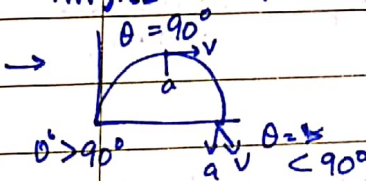
$$= 10000 = 50 J$$



→ Note

K.E & P.E are not equal because it is true that the whole K.E would be converted into potential E but velocity is not fully constant or zero, so it must be calculated.

Angles b/w $\vec{v}$ & $\vec{a}$



→ $\vec{v}$ & $\vec{a}$ are perpendicular only one time at peak.

→ height of projectile

if $\theta = 0^\circ$

$$\sin^2 \theta = 0 \text{ so}$$

$$H = 0$$

if $\theta = 90^\circ$

$$\sin^2 90 = 1$$

$$H = \frac{v_i^2}{2g} = \text{max}$$

if $\theta = 45^\circ$

$$\sin^2 45 = \left(\frac{1}{\sqrt{2}} \right)^2 = \frac{1}{2}$$

$$H = \frac{v_i^2}{2} \times \frac{1}{2}$$

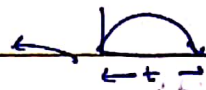
$$h = \frac{h_{\text{max}}}{2}$$

→ if $v_i = \text{constant}$ then $\frac{H_1}{H_2} = \frac{\sin^2 \theta_1}{\sin^2 \theta_2}$ (angle of projection)

$\theta = "$ then $\frac{H_1}{H_2} = \frac{v_{i1}^2}{v_{i2}^2}$ (diff initial velocities)

Day:

Time of flight



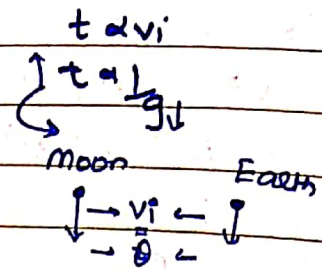
Date: point of projection → point of impact

$$t = \frac{2v_i \sin \theta}{g}$$

→ if $\theta = 0^\circ$ then $t = 0$ ($\sin 0 = 0$)

→ if $\theta = 30^\circ$ then $t = \frac{t_{\max}}{2}$ ($\sin 30 = \frac{1}{2}$)

→ if $\theta = 90^\circ$ then $\sin 90^\circ = 1$ so $t = \frac{2v_i}{g}$



→ time to reach at max height (Time to)

$$t = \frac{2v_i \sin \theta}{g} \times \frac{1}{2}$$

$$= \frac{v_i \sin \theta}{g}$$

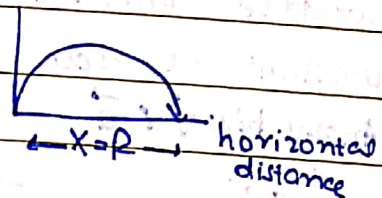
summit

moon has less value of g so it will take more time than Earth to be launched on earth

$t \neq 0$ because

→ Range of Projectile

$$R = \frac{v_i^2}{g} 2 \sin \theta \cos \theta = \frac{v_i^2 \sin 2\theta}{g}$$



→ if $\theta = 0$ $\sin 0 = 0$ so $R = 0$

→ if $\theta = 45^\circ$ $\sin^2 45 = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

→ if $\theta = 45^\circ = \frac{v_i^2}{g} \sin 2(45)^\circ$

$$= \frac{v_i^2}{g} (\sin 90^\circ) = 1$$

$$= R_{\max} = \frac{v_i^2}{g}$$

if $\theta = 90^\circ \rightarrow R = 0 \rightarrow \frac{v_i^2}{g} \sin 2(90)^\circ$

Note:- after 45° the range will start decreasing

Variations

→ $g = \text{constant}$

$$v_i = \text{constant} \quad \frac{R_1}{R_2} = \frac{\sin 2\theta_1}{\sin 2\theta_2}$$

$$\theta = \text{constant} \quad \frac{R_1}{R_2} = \frac{v_i^2}{v_i^2}$$

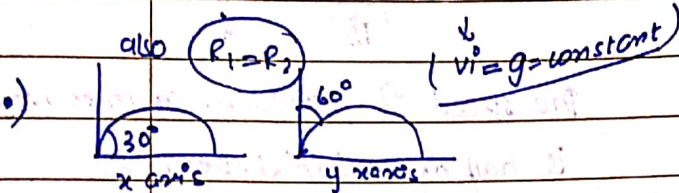
Day: / /

Date: / /

Important Results

-) if $\theta_1 + \theta_2 = 90^\circ$ then $R_1 = R_2$ (one having sum of 90 would have equal ranges)
(if different pairs are given, in a mcq, the complementary angles)

$$30^\circ + 60^\circ = 90^\circ \rightarrow R_1 = R_2$$



if $R = 100m$ $\theta = 45^\circ$

$$\frac{R}{4} = H$$

$$\frac{100}{4} = 25m = H$$

•) $R = nH$ $\tan \theta = 4/n$

e.g if $R = H$ this means that

$$n = 1$$

$$\theta = \tan^{-1} 4/1 = 76^\circ$$

$$0^\circ < \theta < 76^\circ$$

at 76° $R = H$

if $> 76^\circ$ $R < H$

(less) $< 76^\circ$ $R > H$

if $n = 4$

$$R = 4H \rightarrow \theta = \tan^{-1} 4/4 = 45^\circ$$

→ Relation b/w H & time of flight

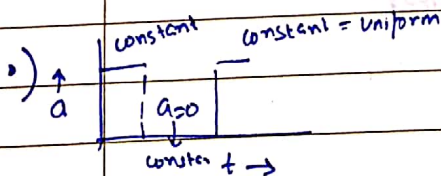
$$\frac{gt^2}{8} = H = \frac{H}{t^2} = \frac{g}{8} = \frac{10}{8} = \frac{5}{4}$$

$$\left(\frac{H}{t^2} = \frac{5}{4} \right) \leftarrow \begin{matrix} H=? \\ t=? \end{matrix}$$

MCQ's

-) Two balls collides elastically such that $m_1 \gg m_2$ where m_2 is at rest = Ans $V_2' = 2V_1$ $V_1' = V$

if $H=?$ $H = \frac{5}{4} t^2$



•) $V_i = 0$

$$V_f = V_i + gt$$

$t = 3\text{sec}$

$h=?$

$$S = V_i t + \frac{1}{2} gt^2$$

$$= 0 + \frac{1}{2} (10) (3)^2$$

$$\frac{9 \cdot 8}{2}$$

Yousaf

Day: / /

Date: / /

•) $F = 400 \text{ N}$

$t = 10 \text{ ms} = 10 \times 10^{-3} \text{ s}$

$I = ?$

$I = F_{av} \times \Delta t$

$400 \times 10 \times 10^{-3}$

$\frac{4000}{1000} = 4 \text{ NS}$

•) $\theta = 15^\circ \rightarrow R$
 $\theta = 75^\circ \rightarrow R = ? R$

$\rightarrow R = nH$

$\theta = \tan^{-1} 4$

$\frac{75}{15}$

$96^\circ \rightarrow R = R$

Past Paper mcd

•) For projectile motion in the absence of air resistance

Ans horizontal acc is zero

•) When $\theta > 45^\circ$ (the range of projection depends upon $\vec{v} \neq \vec{\theta}$)
 at $\theta = 90^\circ$ height is max

$\theta = 45^\circ$ Range is max

Ans) Height will be more but range is less

•) $m_1 \xrightarrow{v_1} \quad \xleftarrow{-v_2} m_2$ opposite direction of velocity (towards each other) same mass

$$m_1 v_1 + m_2 (-v_2) = m_1 v_1' + m_2 (-v_2')$$

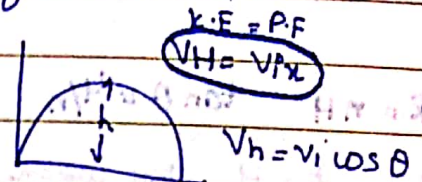
Total momentum after collision

$P = 0 \text{ kgms}^{-1}$

•) $h = \frac{v_i^2 \sin^2 \theta}{2g} \quad L = \frac{L_{max}}{2}$

$\sin 45^\circ = \left(\frac{1}{\sqrt{2}}\right) \times \frac{1}{2}$

The speed of projectile at max height is half of the initial speed v



$V_{H/2} = v_i \cos \theta$

$= \cos \theta = \frac{1}{2}$

$\theta = \cos^{-1} \frac{1}{2}$
 $\boxed{= 60^\circ}$